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No. 302

EFFECT OF SPEED ON ECONOMY OF AIRSHIP TRAFFIC.

By W. Bleistein.

From "Zeitschrift für Flugtechnik und Motorluftschiffahrt," October 28, December 13 and 27, 1924.

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EFFECT OF SPEED ON ECONOMY OF AIRSHIP TRAFFIC.*

By W. Bleistein.

The speed of the vehicle is important in all transportation enterprises. Express trains are operated because the public demands rapid transit, though its cost must be partially paid out of the receipts from the slower traffic. The problem becomes especially difficult on the introduction of a new form of conveyance. There are no results of past experience to be consulted and we can only attempt, through a careful evaluation of all the factors and properties of the new conveyance, to make an approximate preliminary estimate of its availability. The purpose of this article is to consider these factors in the case of an airship.

Although, if we go back to the first works of Schwarz and Count Zeppelin, airships have been in use for 25 years, we find very few available data. The reasons for this are that, before the war, their development was very slow and that, during the war, everything connected with airships was regarded as military secrets and that, after the war, the data collected by airship builders and operators remained unpublished for business reasons. Moreover, many of the data had been obtained for other than commercial purposes. Even the trial trips of the "Bodensee" were made under conditions not applicable to a commercial carrier.

* From "Zeitschrift für Flugtechnik und Motorluftschiffahrt," October 28, December 13 and 27, 1924.

In addition to these more common difficulties due to a lack of statistics, there is another circumstance affecting the solution of speed problems. While, with other vehicles, the actual and relative speed are mostly identical, or currents, which might cause differences between the two speeds, represent known quantities, in the case of an airship we have to take into account the continual changing of the wind in both direction and velocity. Since the velocity of the wind can be quite high with respect to the airspeed of the airship, we must base our solutions of airship economic problems on carefully chosen mean values. This requires the investigation of individual cases. We will accordingly endeavor to show first the connection between the speed and the factors which affect the cost of operation.

Definitions.- Aviation, the most recent branch of technical science, employs many expressions which have not yet been clearly enough defined to avoid ambiguity. It seems best, therefore, to give brief definitions of a few terms which we shall employ in the following investigation.

Lift is the buoyant force of the gas in the hull of an airship under the conditions of the gas and outside air existing at the time. When an airship is in equilibrium, the lift is equal to the "full load" and their algebraic sum is zero.

Ascensional Force is the difference between the lift and the full load, when an airship is not in equilibrium. It becomes negative or discensional, when the lift is less than the full load.

Dead load equals the full load minus the useful load, which, in a commercial airship, consists exclusively of the pay load, i.e., the weight of the passengers and freight.

Weight of Airship equals the dead load minus the weight of the power plants and embraces everything required for the trip (including emergency ballast) and the crew, so far as they do not belong to the power plants.

Weight of Power Plants includes, in addition to the engines and propellers and essential accessories, the power cars (which are regarded simply as engine supports and service platforms) and the part of the crew in charge of the power plants.

The weight of the quarters for the passengers and freight, whether consisting of special cars or of compartments in the hull, may be regarded either as a portion of the dead load or, as in the present article, of the pay load, especially as regards passenger accommodations. The latter course is justified by the fact that greater comfort means greater weight and correspondingly greater charges.

Fuel includes both gasoline and oil for the engines. In weight calculations, the tanks and pipes are often included, since their weight depends on the amount of fuel carried.

Disposable Load is the sum of the fuel load and the pay load.

Fuel reserves are added to the regular fuel supply, since they commonly bear a certain percentage ratio to the latter, while the spare parts belong to the equipment of the hull or power plants.

Radius of Action is the distance the airship can travel at a given airspeed in still air, with full utilization of the disposable load. Hence it is a variable quantity dependent on the speed.

Symbols

H = lift of airship in kg.

V = volume of gas in m^3.

T = weight of airship in kg (without power plants).

M = weight of power plants in kg.

B = weight of fuel in kg (including tanks and pipes).

Q = pay load.

L = disposable load.

N_e = brake horsepower of engines (B.H.P.).

W = drag of airship in kg.

S = flight distance in meters.

A_R = radius of action in meters.

A_D = maximum flight duration in hours.

A_d = maximum flight duration in seconds.

v = airspeed of airship in meters per second.

u = ground-speed of airship in meters per second.

w = wind velocity in meters per second.

α = angle of wind to direction of flight.

β = angle between course steered and route traveled.

η = efficiency of drive and of propeller.

a = lift in kilograms per cubic meter of gas.

b = weight of fuel consumed in kg per HP.-hour.

l = weight in kg of tanks and pipes per 100 kg fuel.

e = $b\,(1 + l/100)$ = mean total weight in kg of fuel consumed per HP.-hour.

t = time in seconds.

Using above symbols, we obtain:

$$u = w \cos\alpha \pm \sqrt{v^2 - w^2 \sin^2\alpha} \text{ (Fig. 1)} \tag{1}$$

$$v = \pm \sqrt{u^2 + w^2 - 2\,u\,w\,\cos\alpha} \text{ (Fig. 1)} \tag{2}$$

$$H = T + M + L \text{ (kg)} \tag{3}$$

$$L = Q + B \text{ (kg)} \tag{4}$$

$$H = a\,V \text{ (kg)} \tag{5}$$

$$N_e = c\,V^{2/3}\,v^3, \text{ good for similar airships} \tag{6}$$

$$N_e = C\,v^3, \text{ good for one and the same airship} \tag{6a}$$

$$B = e\,N_e\,\frac{t}{3600} \tag{7}$$

$$N_e = \frac{vW}{75\eta} \tag{8}$$

PART I.

Economic Service.- By this is meant the airship service for which the public pays. The factors of this service are the loads carried and the distances traveled. Generally the transportation charges are based on the ton-kilometer or passenger-kilometer. The time consumed in a trip has a very indefinite value. The speed of the vehicle is, in a certain sense, prescribed to the user. Sometimes he will pay more and at other times less than the value the speed really has for him. On the other hand, the carrier must adopt a speed which he considers suitable for the traffic on the given line. The economic service is therefore always based on a certain speed. In each instance it is the product of the load times the distance, i.e., the transportation work QS in kg-m at the speed u.

The charges are based on the scheduled speed, but the expenses depend on the airspeed required for maintaining the necessary ground-speed, under the prevailing velocity and direction of the wind. The relation of these quantities is given by equations (1) and (2).

Maximum Economic Service.- Although it by no means follows that the maximum economic service coincides with the maximum economy of operation, it is nevertheless important to determine under what conditions an airship can perform its maximum economic service. For the sake of simplicity, the effect of the wind on the ground-speed will at first be disregarded, thus assuming the ground-speed to be equal to the airspeed.

In the present article, the efficiency is considered constant. This assumption is justifiable in the range of velocity variations occurring in normal traffic, since, in the vicinity of the maximum, at which the work must naturally be done, the efficiency is nearly constant. The object of the experiments is to obtain data for a suitable airship design. We are therefore perfectly free to construct the airship on the basis of such data and to so choose the parts as to attain the maximum efficiency under the assumed conditions.

The airship belongs to the class of vehicles which carry their source of power with them. Hence there is a relationship between B and S, which may be found as follows:

Equation (7), multiplied by v, gives

$$v\ B = e\ N_e \frac{vt}{3600} \tag{9}$$

Since $vt = S$, we have

$$S = \frac{v\ B\ 3600}{e\ N_e} \tag{10}$$

and, taking equation (8) into consideration, we have

$$S = \frac{75 \times 3600\ B\ \eta}{e\ W} \tag{11}$$

By substituting this value in equation (9) and simultaneously writing $Q = L - B$ (eq. (4)), we have

$$Q\ S = \frac{75 \times 3600\ B\ (L - B)\eta}{e\ W} \tag{12}$$

Since uniformity of speed is assumed, L, e, η and W are constant and the economic service is maximum, when

$$\frac{d(Q\ S)}{d\ B} = 0.$$

Hence

$$B = \frac{1}{2} L \tag{13}$$

The maximum economic service is therefore attained when the weight of the fuel equals half the disposable load. Since, however, the radius of action equals the distance which can be traveled when the fuel constitutes the entire disposable load, it follows that the maximum economic service is attained when the transporting distance equals half the radius of action.

This rule is, in a certain sense, fundamental, since equation (13) shows that the airspeed v does not affect its validity. It is not even affected by the strength and direction of the wind, if the calculations are based on the mean velocity for the whole distance.

Radius of Action.- If the radius of action of an airship at a given speed is known, its economic service can then be directly determined for any distance. The quantity of fuel $B = L$ corresponds to the radius of action. If the distance is shortened, less fuel is required and the pay load Q can be increased by the same amount. If we express the distance S in percentage of the radius of action and the actual pay load in percentage of its maximum, we obtain Table I and Fig. 2.

This curve is valid for any airship and any radius of action at the corresponding speed and shows what percentage of the maximum pay load can be transported any desired distance. The correctness of this statement follows from the following considerations.

The radius of action is the product of the speed and the maximum duration, hence $A_R = A_d\ v$ (14).

Since, for the radius of action, the weight of the fuel equals the total disposable load, the equation

$$B = \frac{e\ N_e\ t}{3600} = \frac{e\ C\ v^3\ t}{3600}$$

changes into the form

$$L = \frac{e\ C\ v^3\ A_d}{3600}$$

and becomes

$$A_R = \frac{3600\ L}{e\ C\ v^2} \tag{15}$$

Table I.

S	Q	QS	S % of A_R	QS % of $(QS)_{max}$
0	2	0	0	0
0.1	1.9	0.19	5	19
0.2	1.8	0.36	10	36
0.3	1.7	0.51	15	51
0.4	1.6	0.64	20	64
0.5	1.5	0.75	25	75
0.6	1.4	0.84	30	84
0.7	1.3	0.91	35	91
0.8	1.2	0.96	40	96
0.9	1.1	0.99	45	99
1	1	1	50	100
1.1	0.9	0.99	55	99
1.2	0.8	0.96	60	96
1.3	0.7	0.91	65	91
1.4	0.6	0.84	70	84
1.5	0.5	0.75	75	75
1.6	0.4	0.64	80	64
1.7	0.3	0.51	85	51
1.8	0.2	0.36	90	36
1.9	0.1	0.19	95	19
2	0	0	100	0

Since L, e and C are constants of the airship under consideration, the proportion for another radius of action A_{R_1} with its corresponding speed v_1 will read $A_R : A_{R_1} = v_1^2 : v^2$

or $A_R\, v^2$ is a constant (Fig. 3)

(16)

Hence the law that the radii of action are inversely proportional to the square of the corresponding speeds.*

The transportation distance corresponding to the maximum eco-

* For any given airship, the upper portion of the curve is limited by the magnitude of the maximum speed and it becomes necessary to consider the changeableness of the efficiency at the lower speeds.

nomic service for the chosen speed v_1, then becomes

$$S_1 = \frac{1}{2} \frac{V^2}{v_1^2} A_R \tag{17}$$

If, for a given speed v_1, any given distance S_2 is greater or less than S_1, the corresponding work of transportation is

$$Q_2 \ S_2 = \frac{S_2}{S_1} \left(2 - \frac{S_2}{S_1}\right) Q_1 \ S_1 \tag{18}$$

in which the symbols with the suffix $_1$ are the symbols corresponding to the maximum economic service. Equation (18) corresponds to a parabola of the form $y = 2x - x^2$, in which $y = Q_2 S_2 : Q_1 S_1$ and $x = S_2 : S_1$. If, as in Fig. 2, the distance S_2 is expressed in percentages of the radius of action, the parabola passes over to the form $y = 4x - 4x^2$.

The importance of the radius of action renders it desirable to be able to find its relation to the speed from the structural data of the airship. As already mentioned, its magnitude depends on the available amount of fuel, which, according to equation (3), is

$$B = L = H - T - M \tag{19}$$

If the weight of the power plants, with reference to the brake horsepower of the engines, is designated by i,

$$i = \frac{M}{M_n} \ (\text{kg/HP.}) \tag{20}$$

and

$$H - T = K \tag{21}$$

then, by employing equations (19) and (20), we obtain

$$L = K - i \ N_e \tag{22}$$

The fuel consumption per second is represented by $e\ N_e/3600$. The flight duration is then

$$A_d = \frac{3600\ (K - i\ N_e)}{e\ N_e} \tag{23}$$

or, by employing equation (6a),

$$A_d = \frac{3600\ (K - i\ C\ v^3)}{e\ C\ v^3} \tag{24}$$

According to equation (15), the maximum transportation work E is then

$$E = \frac{1}{2}\ A_R\ \frac{1}{2}\ L = \frac{3600}{4}\ \frac{(K - i\ C\ v^3)}{e\ C\ v^2}\ (K - i\ C\ v^3)$$

$$E = \frac{900\ (K - i\ C\ v^3)^2}{e\ C\ v^2} \tag{25}$$

Speed of Minimum Fuel Consumption.- While the previous deductions were for the purpose of discovering under what conditions the maximum service can be gotten from an airship, i.e., how to get the maximum income, we must now try to learn under what conditions the expenses can be reduced to the minimum. The operating expenses of an airship consist chiefly of the expenditures for gas and fuel (including oil). The gas consumption is dependent on the permeability of the gas bags and on the flight altitude. In a certain sense the flight altitude depends on the fuel consumption, at least in the absense of any device for providing ballast during flight, and the fuel consumption is, in turn, dependent on the speed. In connection with the present investigation, it will therefore be necessary to determine only at what speed the fuel consumption is smallest, in order to be simultaneously convinced that the gas consumption, in so far as it is effected by the fuel consumption, is subject to the same conditions. (See C. Wieselsberger, "Zeitschrift für Flugtechnik und Motorluftschiffahrt," 1913, No. 2, and W. Bleistein, "Z.F.& M." No. 5, 1924.)

The fuel consumption per time unit, according to equations (6a) and (7), is $e\ C\ v^3/3600$ and per distance unit it is $e\ C\ v^3/3600\ u$. This expression becomes a minimum for

$$\frac{d\,\dfrac{e\ C\ v^3}{3600\ u}}{dv} = 0 \qquad (26)$$

If, with the aid of equation (1), u is replaced by v, w and α, equation (26) becomes

$$\frac{d\left[\dfrac{e\ C\ v^3}{3600\ (w\ \cos\alpha \pm \sqrt{v^2 - w^2\ \sin^2\alpha})}\right]}{d\ v} = 0$$

and we find, as the expression for the minimum fuel consumption per time unit,

$$v_{ec} = w\sqrt{\frac{3}{8}\left(4 - \cos^2\alpha - \cos\alpha\sqrt{8 + \cos^2\alpha}\right)} \qquad (27)$$

For the application of this equation, it is best to return to the value v_{ec}/w, which, in Fig. 4, is plotted against the angle α.

The speed, according to equation (27), can be designated as an economical speed, but only with exclusive reference to the gas and fuel consumption. It depends only on the direction and strength of the wind. When there is no wind, there is no speed of minimum fuel consumption, since $v_{ec.}$ then equals zero and every speed increase necessitates increased fuel consumption. This conclusion follows from equation (7), since, with $u = v$, the consumption per unit distance becomes $e\ C\ v^2/3600$. Also with the wind in the flight direction $(\alpha = 0^\circ)$, $v_{ec.} = 0$. With a head wind $(\alpha = 180^\circ)$, $v_{ec.} = 1.5\ w$, so that the speed of minimum fuel consumption is equal to one and a half times the wind velocity.

On account of the continual variations of w and α, an airship will have on every trip a different speed of minimum fuel consumption. Even during the same flight, $v_{ec.}$ can vary considerably. We would be led to wrong conclusions if we should combine a mean value of this speed with the maximum economic service in order to arrive at the maximum economy of operation. Part II will show what effect the speed of minimum fuel consumption really has on the cost of traffic enterprises.

Gas Consumption.- It has already been mentioned that the consumption of the lifting gas depends chiefly on the flight altitude. Increased speed increases the fuel consumption, thus more rapidly lightening the load and increasing the ascensional force of the airship, which causes it to rise until it reaches the altitude of maximum internal pressure, where gas is released and equilibrium is restored by the loss in lift. There are, however, ways of reducing this loss. We have already mentioned the obtaining of water ballast from the exhaust gases. Though theoretically easy, this method has not yet been perfected. It is also possible to hold an airship dynamically, by means of the elevator, at the desired altitude. This method is, however, seldom and reluctantly resorted to, because a so-called "light" airship steers badly and loses speed on account of its inclined position. This loss of gas is proportional to the fuel consumption and amounts to B/a (lift per m^3) for the whole flight.

It is also probable that the pressure variations, existing on the surface of the airship during flight and whose magnitude depends on the speed, affect the gas losses through the permeability of the gas bags. No experimental tests of this have been made but the effect is certainly so slight, in comparison with the gas losses resulting from the fuel consumption, that it may be disregarded.

Crew.- The weight of the crew and their quarters is considered as being included in the dead load. In general, the size of the crew depends on the size of the airship. Any accurate rule can hardly be formulated to estimate these costs for a single trip. The crew number does not vary much for a given size of airship. It depends more on whether a single, double, or triple watch is required, according to the length of the voyage. The number of attendants for the engines depends on the number of engines, rather than on the total horsepower.

For determining the operation expenses, the question still remains open, according to the size of the airship, for upon this depends the original purchase price, the amortization, the value of the plant, upkeep, reparations, etc. Hence, it is always desirable to get along with as small an airship as possible and it is all the more important to determine the relation between speed and size.

Size of Airship.- The foregoing deductions apply to airships of any size, while assuming that the engine power is always sufficient for attaining the maximum economic service and the desired speed. Since, theoretically, the size of an airship can be reduced until its lift only just equals its dead load, so that it has no carrying capacity, and since, on the other hand, with increase in the size of an airship, the engine power and fuel consumption, necessary to attain a certain speed, increase, we can not be satisfied with the foregoing relations. It is quite conceivable that a carrying capacity, which is the best for a given airship, may be attained by another airship or by the same airship under more favorable conditions, without being the best. In order to clarify these relations, we must determine what effect the speed has on the size and weight of the airship.

The lift must equal the combined weights of the airship proper, the power plants, fuel and carrying capacity, i.e., the "full load." Hence $H = T + M + B + Q$, all the members of this expression being dependent on the two variables V and v.

The value of T can be determined only by experiment and is difficult to obtain. It may be assumed that the weight of the airship proper varies from approximately as the 2/3 power of its volume up to its full volume. As we shall show later, the value $T = k\ V^{4/5}$ is sufficiently accurate for the present investigation. The expression for M can, with the help of equations (6) and (20) be reduced to the form

$$M = i\ e\ V^{2/3}\ v^3 \qquad (30)$$

If the flight distance is given, B can likewise be replaced by an expression containing only V and v as unknown quantities. From $B = e\ N_e t/3600$ we obtain, for no wind with $t = S/v$

$$B = \frac{e\ c\ V^{2/3}\ v^2\ S}{3600} \qquad (31)$$

If we assume that an airship is to be designed for the maximum carrying capacity, Q must equal B. Then, since $H = a\ V$, equation (29) becomes

$$a\ V = k\ V^{4/5} + i\ c\ V^{2/3}\ v^3 + \frac{2\ e\ c\ V^{2/3}\ v^2\ S}{3600} \qquad (32)$$

If, for further simplification, we adopt $T = k\ V^{2/3}$ instead of $T = k_1 V^{4/5}$, we then obtain the very simple form

$$V = \frac{1}{a^3}\left(k_1 + i\ c\ v^3 + \frac{e\ c\ v^2 S}{1800}\right)^3 \qquad (32a)$$

If no consideration be given the maximum carrying capacity (i.e., if Q may be either less or greater than B), then no simple form, after the manner of equation (32a), is possible and we are compelled to find, by certain expedients, the value of V from the equation

$$a\ V = k\ V^{4/5} + i\ c\ V^{2/3} v + \frac{e\ c\ V^{2/3} v^2\ S}{3600} + Q \qquad (33)$$

We must once again call attention to the fact that equations (32), (32a) and (33) can be used only for rough calculations, such as are allowable for preliminary calculations to determine whether an enterprise is likely to be profitable. We must understand that the exponent of the first expression and the coefficients depend entirely on the designer. In what follows, it will therefore be necessary to consider more carefully all the coefficients thus far employed.

Coefficients. a = coefficient of lift (kg/m³).- The coefficient a is equal to the lifting force, in kg, of a cubic meter of gas. It is largely dependent on the temperature of the gas and air, the barometric pressure, the gas density and humidity. With the symbols adopted by Prof. Eberhardt, its value is

$$a = \frac{b}{2.1525}\left(1 - 0.377\,\frac{b_o}{b} - s\,\frac{T}{T_1}\right) \qquad (34)$$

(See Prof. C. Eberhardt, "Luftschiffahrt" and Prof. R. Emden, "Grundlagen der Ballonfuhrung.")

Hence it is very changeable and, in order to determine the suitable type of airship, we must know, as accurately as possible, the mean values of the factors, namely the temperature of the gas and air, the barometric pressure, the gas density and the humidity. As practical computation values for good gas at sea-level, we can adopt for

Hydrogen, $a_w = 1.16 - 1.17\ \text{kg/m}^3$,

Helium, $a_h = 1.07 - 1.08\ \text{kg/m}^3$.

b = fuel consumption in kg/HP.-hr.- The specific fuel consumption is a characteristic of each engine. It is desirable for it to be as small as possible for the R.P.M. and HP. and air density constituting the working conditions of the engine. The minimum fuel consumption of present-day engines is about 180 g per HP.-hr. and the minimum oil consumption is about 8 g per HP.-hr. For approximate calculations, we may therefore employ $b = 0.2$ kg/HP.-hr.

c = coefficient of drag ($kg \times s^2/m^4$).- This coefficient, with the aid of equations (6) and (8), becomes $c = \xi \gamma / 75 \eta g$. The expression $\xi \gamma / g$ (See "Hütte" 22, I, 359) was combined with the factor $1/75\eta$, because all printed airship data always give only the B.HP. of the engines, but not the efficiecy of the drive and of the propeller. Hence it seemed better to find the unknown quantity c as a whole and subsequently to use it, than to increase the inaccuracy by splitting it up into factors. From the published data on Schutte-Lanz and Zeppelin airships we obtain, from $N = c\ V^{2/3}\ v^3$, a value $c = 3 \times 10^{-5}$ for modern types (Table II). For $\eta = 0.75$ this would give a ξ of about 0.0127. This value is naturally larger than published drag coefficients of airship models, because it includes the whole airship with tail unit, cars, etc., while wind tunnel experiments are generally performed with hull models only.

Table II.

Type	V m^3	Ne HP.	v m/s	$V^{2/3}$	v^3	$V^{2/3} v^3$ 10^{-5}	c 10^{-5}
k	20800	540	20.5	756	8615	6.5	8.31
l	22140	540	20	790	8000	6.32	8.55
m	22470	630	23.4	796	12813	10.2	6.18
n	25000	630	22.5	855	11391	9.74	6.47
o	24900	630	23.6	853	13144	11.21	5.62
p	31900	840	26.7	1006	19034	19.15	4.39
q	35800	960	26.5	1086	18610	20.2	4.86
r	55200	1440	28.7	1450	23640	34.28	4.20
s	55500	1200	27.7	1457	21254	30.97	3.87
t	55800	1200	28.9	1460	24138	35.24	3.41
u	55800	1200	29.9	1460	26731	39.03	3.07
v	56000	1200	30.2	1464	27544	40.32	2.98
w	68500	1200	28.6	1674	23394	39.16	3.86
x	62200	2030	36.4	1570	48229	75.72	2.68
y	20000	960	36.8	737	49836	36.73	2.61
z	22550	960	35.4	798	44361	35.41	2.71
b	25000	740	24.5	855	14710	12.57	5.88
c	32500	840	23.6	1018	13140	13.39	6.28
d	35100	840	25.9	1072	17370	18.63	4.51
e	38800	960	26.9	1146	19465	22.31	4.30
f	56350	1200	28.5	1470	23150	34.02	3.53
g	63800	2240	34	1597	39300	62.76	3.57
h	78000	2240	32.5	1821	34330	65.51	3.58

Types k to z, Zeppelin airships; b to h, Schutte-Lanz airships.

The magnitude of c is subject to great fluctuations. It is affected by the shape and nature of the surface of the airship hull, by the number and size of the tail planes, by the arrangement and nature of the cars, etc. In calculating c from the data of actual airships, allowance must be made for the fact that the airship volume is always given as the gas volume, while the drag refers to the outer volume. In similar airships, however, the outer volume may be regarded as a constant multiple of the gas volume, without great error. In Table II this constant is already contained in c.

e = gross fuel consumption in kg/HP.-hr.- This value is composed of the weight of the fuel consumed per HP.-hour plus the corresponding fraction of the weight of the tanks and piping for each 100 kg of fuel (See later under l), so that $e = b\ (1 + l/100)$, 0.214 kg/HP.-hr. being taken as the value of e.

i = weight coefficient of the power plant in kg/HP. (See equation (20)).- The value $i = M : N_e$ is naturally not constant. Aside from the type and quality of the engine, it depends on whether the total power is obtained through large or small engines, through many or few power units. The following values were obtained from actual and computed data.

Table III

N	500	1200	2400	3000	3900 HP.
i	5.1	4.4	4.3	4	3.95 kg/HP.

Although, in these values, no consideration was given to uniformity with respect to the power of the individual engines or their number, they nevertheless give, as plotted in Fig. 5, a quite regular curve, which seems to justify the claim that the weight of the engines, with respect to their power, increases as the total power decreases. For subsequent calculations, the value $i = 4$ kg/HP. was taken.

k = weight coefficient of the airship hull ($kg/m^{12/5}$).- The determination of the weight coefficient for the hull of the airship encounters the greatest difficulties. Here scientific progress, the constructor's knowledge,, the type of airship, the safety factor, in short, so many factors are involved that conclusions can be drawn from published data only with the greatest discretion. We never know how much the weights are affected by the volume of the airship. If we trace, from the calculated numbers given in Table IV, the line $T = f(V)$ (Fig. 6), we can draw from it no absolute conclusion. It is obvious, however, that the first airships of a building period are more difficult to construct than subsequent ones, due to less experience. It can be confidently claimed only that the weight increases somewhat more slowly than the volume. Likewise, pure mathematical calculations can give only the approximate conclusion that the weight changes somewhat less tham in proportion to the volume, but apparently somewhat more than proportional to the $^{2/3}$ power of the volume. If the actual change has ever been determined, it has been kept secret by the construction firms. In order to proceed farther, it was assumed that the weight of the hull is proportional to $V^{4/5}$. This probably approximates the actual weight of airships of the sizes entering into the problem. Even with this assumption, however, we cannot proceed without reservations. Data on individual weights are nowhere to be found. Even the total weights of the airships must be found by subtracting the useful load from the lift. After this is done, the weight of the hull must be found by assuming the correctness of the coefficient i (Table IV). The value of k is then obtained from $a\,V = L + k\,V^{4/5} + M$, according to equation (32):

$$k = \frac{a\ V - L - M}{V^{4/5}} \tag{35}$$

In subsequent calculations it was assumed that $k = 3 \left(\frac{kg}{(m^3)^{4/5}}\right)$

Table IV

Type[1]	Year built	V m^3	N HP.	L kg	M[2] kg	H[3] kg	T[4] kg	k[5] $kg/m^{12/5}$
k	1913	20800	540	8800	2750	24340	12790	4.492
l	1914	22140	540	8850	2750	25900	14300	4.777
m	1914	22470	630	9200	3150	26290	13940	4.603
n	1914	25000	630	12200	3150	29250	13900	4.214
o	1915	24900	630	11100	3150	29130	14880	4.525
p	1915	31900	840	16200	3220	37320	17900	4.465
q	1915	35800	960	17900	4520	41890	19470	4.426
r	1916	55200	1440	32500	6410	64580	25670	4.128
s	1917	55500	1200	36400	5520	64940	23020	3.687
t	1917	55800	1200	37800	5520	65290	21970	3.504
u	1917	55800	1200	39000	5520	65290	20770	3.312
v	1917	56000	1200	40000	5520	65520	20000	3.180
w	1917	68500	1200	52100	5520	80140	22520	3.041
x	1918	62200	2030	44500	8630	72770	19640	2.871
y	1919	20000	960	10000	4520	23400	7880[6]	2.856
z	1921	22550	960	11200	4520	26320	9100[6]	2.996
b	1914	25000	740	7900	3590	29250	17760	5.384
c	1915	32500	840	14000	4030	38020	19990	4.913
d	1915	35100	840	15700	4030	41070	21340	4.931
e	1916	38800	960	21500	4510	45400	19390	4.135
f	1917	56350	1200	37500	5520	65930	22910	3.625
g	7)	63800	2240	46100	9410	74650	19140	2.742
h	7)	78000	2240	59500	9410	91260	22350	2.727

1) As published in "Zeitschrift für Flugtechnik und Motorluftschiffahrt," only airships after 1912 being taken.

2) M is computed from $M = i\ N_e$, wherein i is taken from Fig. 5.

3) $H = a\ V$, wherein $a = 1.17\ kg/m^3$.

4) $T = H - M - L$.

5) $k = T : V^{4/5}$.

6) and 7) See next page.

l = weight coefficient of fuel tanks and pipes.- According to completed structures, we can calculate on about 7 kg for tanks and pipes for every 100 kg of fuel. Hence $l = 0.07$ kg per kg of fuel.

Connection between v and V.- With the given bases for the coefficients, we can proceed to the further consideration of equation (33).

From the weight balance (equation (29)), $H = T + M + B + Q$, an expression was obtained by transformation, which practically contained the magnitude and speed of the airship. It was

$$a\ V = k\ V^{4/5} + i\ c\ V^{2/3} v^3 + \frac{e\ c\ V^{2/3}\ S\ v^2}{3600} + Q.$$

If we make the last two terms of the right side, which represent the fuel and pay load, equal to zero, then the equation represents an airship whose total lift is entirely absorbed by the weight of the airship and power plants. From this we accordingly obtain the theoretical maximum speed for each airship volume.

$$a\ V = k\ V^{4/5} + i\ c\ V^{2/3} v^3$$

$$v^3 = \frac{a\ V^{1/3} - k\ V^{2/15}}{i\ c} \qquad (36)$$

$$v = \sqrt[3]{\frac{a\ V^{1/3} - k\ V^{2/15}}{i\ c}} \qquad (37)$$

the latter being the upper speed limit of an airship of given size.

* (Cont. from Page 23.)

6) Reduced 50 kg per passenger, on account of passenger arrangements.
7) Destroyed before completion, in fulfillment of treaty stipulations.

This value is of interest in so far as we might wish to know the range of possible speeds in enterprises with airships of a given size.

The numbers in Table V and the curves in Fig. 7 were obtained with $a = 1.17$ kg/m³, $k = 3$ kg/m$^{12/5}$, $i = 4$ kg/HP., $c = 3$ (10^{-5} kg s² m⁴).

Table V.

V	$aV^{1/3}$	$kV^{2/15}$	$aV^{1/3} - kV^{2/15}$	$\frac{aV^{1/3} - kV^{2/15}}{i\ c}$	$v = \sqrt[3]{\frac{aV^{1/3} - kV^{2/15}}{i\ c}}$ m/s
20000	31.761	10.980	20.781	173200	55.7
40000	40.014	12.323	27.691	230800	61.3
60000	45.804	13.008	32.796	273300	64.9
80000	50.414	13.517	36.897	307400	67.5
100000	54.308	13.925	40.382	336500	69.6
120000	57.710	14.267	43.443	362000	71.3
140000	60.753	14.564	46.189	384900	72.7
160000	63.519	14.825	48.694	405800	74.0
180000	66.062	15.060	51.002	425000	75.2
200000	68.423	15.273	53.150	442900	76.2
250000	73.688	15.734	57.954	483000	78.5
300000	78.325	16.121	62.204	519200	80.4

We see that the speed limits are about twice as high as the actual maximum speeds of modern airships.

Speed limits for given airship sizes and distances.- Continuing on the given line, we come next to the question of the maximum speed for a given distance, without reserving any of the carrying capacity for pay load, as in the case of a military observation airship. With $Q = 0$, equation (33) becomes

$$aV = kV^{4/5} + i\ c\ V^{2/3}\ v^3 + \frac{e\ c\ V^{2/3}\ v^2\ S}{3600}.$$

By transforming, we obtain

$$v^3 + \frac{e\ S\ v^2}{3600\ i} = \frac{a\ V^{1/3} - k\ V^{2/15}}{i\ c} \qquad (38)$$

This equation is best solved graphically, as follows: Each expression contains the dimension m^3/s^3. If we introduce into a diagram in the ordinate scale m^3/s^3 the expression of the right side as a function of V and the sum of the expressions of the left side of the equation as a function of v, the corresponding values will then fall on the same ordinate. This is most clearly done by drawing the curve of the function of V on one side of the ordinate axis and the curves of the function of v for different values of S on the other side. Thus we obtain Fig. 8 with the values of Table VI, wherein the values for $\frac{a\ V^{1/2} - k\ V^{2/15}}{i\ c}$ are taken from Table V.

Table VI.

Values for equation (38). - Determination of $v^3 + \frac{e\ S\ v^2}{3600\ i}$

v	v^3	$\frac{e\ S\ v^2}{3600\ i}$ S=1000	$v^3+\frac{e\ S\ v^2}{3600\ i}$ S=1000	S=2000	S=3000	S=4000	S=5000
5	125	434	559	993	1427	1861	2295
10	1000	1736	2736	4472	6208	7944	9680
15	3375	3906	7281	11187	15093	18999	22905
20	8000	6944	14944	21889	28834	35778	42722
25	15625	10851	26476	37327	48178	59029	69880
30	27000	15625	42625	58250	73875	89500	105125
35	42875	21268	64143	85411	106679	127947	149215
40	64000	27778	91778	119556	147334	175112	202890
45	91125	35157	126282	161439	196596	231753	266910
50	125000	43403	168403	211806	255209	298612	342015

v	v^3	S=6000	S=7000	S=8000	S=9000	S=10000
5	125	2729	3163	3597	4031	4465
10	1000	11416	13152	14888	16624	18360
15	3375	26811	30717	34623	38529	42435
20	8000	49667	56612	63556	70500	77445
25	15625	80731	91582	102433	113284	124135
30	27000	120750	136375	152000	167625	183250
35	42875	170483	191751	213019	234287	255555
40	64000	230668	258446	286224	314002	341780
45	91125	302067	337224	372381	407538	442695
50	125000	385418	428821	472224	515627	559030

Here also the same values as before were adopted for a, k, i and c, the value chosen for e being 0.25 kg per Hp.-hr. The value of e is greater than the value previously given (0.214 kg/HP.-hr.), in order to allow for adequate reserves. The curves were computed for values of S from 1000 to 10000 km.

If, for example, we desire to find the maximum velocity of an airship of V = 50000 m^3 with a radius of action of 5000 km, we follow the abscissa $V = 50000$ to its intersection with the curve of the V function and the corresponding ordinate to its intersection with the curve of the v function for $S = 5000$ km. The abscissa belonging to this point gives the maximum speed $v = 41.7$ m/s (about).

Interdependence of size of airship, maximum speed, distance and pay load.- The last investigation in this series is intended to determine the maximum speed for airships of different sizes, when the distances and pay load are given. If this investigation is successful, we will have a basis for answering the most important question of such a traffic enterprise, since the determination of the maximum speed for a given size of airship is synonymous with the determination of the minimum size of airship for a given speed. We thus obtain the necessary information for reducing the capital investment to a minimum.

Also the unabridged equation

$$a\,V = k\,V^{4/5} + i\,c\,V^{2/3}\,v^3 + \frac{e\,c\,V^{2/3}\,v^2\,S}{3600} + Q \qquad (33)$$

can be best solved graphically. If the transformation is made in the same manner as before, we obtain

$$v^3 + \frac{e\,S\,v^2}{3600\,i} = \frac{a\,V^{1/3} - k\,V^{2/15}}{i\,c} - \frac{Q}{i\,c\,V^{2/3}} \qquad (39)$$

Fig. 8 is changed only in so far as we replace the former curve for the V function by a set of curves determined by the value of Q.

Table VII and Fig. 9 were made with the values previously employed, in which Q was given a value of 1000 to 50000 kg. The corresponding values again lie on the same ordinates and the method of finding desired values is the same as previously described.

Moreover, the distance between the curve for $S = 0$ and the curve for any desired value of S serves as the scale for correct interpolation and extrapolation. This answers for Q.

Table VII.

V	$\frac{aV^{1/3} - kV^{2/15}}{i\ c}$	$\frac{Q}{i\ c\ V^{2/3}}$ (m^3/S^3) for Q = (kg).					
m^3		1000	2000	5000	8000	10000	12000
20000	173175	11310	22620	56550	90480	113100	135720
40000	230758	7125	14250	35625	57000	71250	85500
60000	273300	5437	10874	27185	43496	54372	65244
80000	307381	4488	8975	22440	35904	44885	53856
100000	336517	3868	7736	19340	20944	38680	46416
120000	362025	3425	6850	17125	27400	34253	41100
140000	384908	3091	6182	15455	24728	30907	37092
160000	405783	2827	5654	14135	22616	28275	33924
180000	425017	2614	5228	13070	20912	26139	31368
200000	442818	2437	4874	12185	19496	24367	29244
250000	482950	2100	4200	10500	16800	20999	25200
300000	519200	1860	3720	9300	14880	18595	22320

Table VII (Cont.)

V m^3	$\frac{aV^{1/3} - kV^{2/15}}{i\ c}$	$\frac{Q}{i\ c\ V^{2/3}}$ (m^3/S^3) for Q = (kg).					
		15000	20000	25000	30000	40000	50000
20000	173175	169660	226200	282750	339300	452400	565500
40000	230758	106875	142500	178125	213750	285000	356250
60000	273300	81555	108740	135925	163110	217480	271850
80000	307381	67320	89760	112200	134640	179520	224400
100000	336517	58020	77360	96700	116040	154720	193400
120000	362025	51375	68500	85625	102750	137000	171250
140000	384908	46365	61820	77275	92730	123640	154550
160000	405783	42405	56540	70675	84810	113080	141350
180000	425017	39210	52280	65350	78420	104560	130700
200000	442818	36555	48740	60925	73110	97480	121850
250000	482950	31500	42000	52500	63000	84000	105000
300000	519200	27900	37200	46500	55800	74400	93000

Table VII (Cont.)

$V\ m^3$	$\frac{a\ V^{1/3} - k\ V^{2/15}}{i\ c} - \frac{Q}{i\ c\ V^{2/3}}\ (m^3/S^3)$ for Q = (kg).					
	1000	2000	5000	8000	10000	12000
20000	161865	150555	126625	82695	60075	37455
40000	223633	216508	195133	173758	159508	145258
60000	267863	262426	246115	229804	218928	208056
80000	302893	298405	284941	271477	262496	243525
100000	332649	328781	317177	305573	297837	290101
120000	358600	355175	344900	334625	327772	320925
140000	381817	378726	369453	360180	354001	347816
160000	402956	400129	391648	383167	377508	371859
180000	422403	419789	411947	404105	398878	393649
200000	440381	437944	430633	423322	418451	413574
250000	480850	478750	472450	466150	461951	457750
300000	517340	515480	509900	504320	500605	496880

$V\ m^3$	15000	20000	25000	30000	40000	50000
20000	3525					
40000	123883	88258	52633	17008		
60000	191745	164560	137375	110190	55820	1450
80000	240061	217621	195181	172741	127861	82981
100000	278497	259157	239817	220477	181797	143117
120000	310650	293525	276400	259275	225025	190775
140000	338543	323088	307633	292178	261268	230358
160000	363378	349243	335108	320973	292703	264433
180000	385807	372737	359667	346597	320457	294317
200000	406263	394078	381893	369708	345338	320968
250000	451450	440950	430450	419950	398950	377950
300000	491300	482000	472706	463400	444800	426200

Relation between V and v, when S and Q are given.- In many instances S and Q are to be regarded as given quantities, i.e., the main object is to carry a given load a given distance. The question then arises as to the relation between V and v. This is answered by a curve which can be constructed from Fig. 9.

Supposing that the curve $V = f(v)$ for $Q = 50000$ kg (or 50 tons) and $S = 10000$ km. We utilize the corresponding points of the curves for $Q = 50000$ kg and $S = 10000$ km and introduce them best in a special diagram as $V = f(v)$. Fig. 10 gives both the original curves

$$I = \frac{a\,V^{1/3} - k\,V^{2/15}}{i\,c} - \frac{Q}{i\,c\,V^{2/3}} \quad \text{for } Q = 50000 \text{ kg}$$

$$II = v^3 + \frac{e\,S}{3600\,i}\,v^2 \quad \text{for } S = 10000 \text{ km}$$

and the desired curve

$$III = V = f(v) \quad \text{for } Q = 50000 \text{ kg and } S = 10000 \text{ km}$$

The previous investigations served to obtain from the characteristics of the airship the bases for determining the effect of speed on the profitableness of an airship traffic enterprise.

It was demonstrated:

1. That the maximum economic service is attainable for a distance of half the radius of action;

2. That the radii of action of an airship are inversely proportional to the squares of its speeds;

3. That there is a speed of minimum fuel consumption only when the airship is in an air current whose direction is not identical with the course flown;

4. That the size of the airship for the desired service depends on the speed;

5. That there is a maximum speed for every size of airship;

6. That among the four quantities V_{min}, v_{max}, S and Q, any three are optional and these determine the value of the fourth.

Part II of this article will attempt to combine these results in such a way as to obtain the maximum profit.

PART II.

Profitableness.- Maximum economy of a traffic enterprise is characterized by the fact that the difference between receipts and expenditures is the maximum. Hence we must always endeavor either to increase the receipts or reduce the expenses, or to do both things at the same time.

The factors affecting the receipts vary but little, after the line has been established and the daily pay load determined. It is not the duty of the engineer to provide for the actual attainment of the expected volume of traffic. All that he can do is to place in the hands of the operator a vehicle fulfilling the requirements. The fare price and freight rate multiplied by the transportation work QS gives the year's income. It is, of course, assumed that the estimate of these values is correctly made in advance.

The question of expenditures is a different one. Here it is the task of the engineer to deliver a vehicle that shall meet all the requirements with the least yearly outlay. In this connection the question of maximum profit becomes a question of minimum outlay.

We will attempt to show how, in consideration of the speed, the suitable airship can be found for a given Q and S.

Speed and saving of time.- It would take us too far to follow out, in all its details, the effect of the speed of the airships on the profitableness of an enterprise. It is only necessary to consider the essential points for rendering such an enterprise profitable. Here belongs, first of all, the effect of the speed on the traffic schedule between two places and the consequent effect on the requisite capital.

The traveler, for whom the voyage is not an object in itself, always desires the greatest possible speed. The owner, on the contrary, on account of the high cost, always prefers the lowest speed consistent with satisfactory frequency. This leads to the consideration of competitive enterprises between airships and railroads or steamboats. (Airplanes are not competitors, because airplanes and airships, on account of their special characteristics, really serve to supplement one another). Speed in itself is not the most important thing for the traveler, but only the saving in time or, more correctly, the diminution of the time lost. If, for example, a train running at the rate of 75 km per hour requires 48 hours, including stops and detours, to travel between two places A and B, 2500 km apart, the traveler is not concerned with the hourly speed but only with the time loss of 48 hours. If an airship should carry him over the same space in 24 hours, there would be a time saving of 24 hours. Even this statement, however, is not absolutely correct. For travelers in general, a saving of an hour by day is worth considerably more than by night. If both conveyances should leave the place A at the same time of an evening and both reach B in the evening, the train in two days and the airship in one day, there would be a utilizable daytime saving of only 12 hours, not of 24 hours. With very few exceptions, the 24 hours saved can be fully utilized only by a person who wishes to make a special connection at B for continuing his journey the same evening. If the connecting vehicle, however, again requires 24 hours to reach the place C and if, in addition to the evening connection, there is also a morning connection, there is again nothing gained as compared with a 36 hour trip from A to B. The traffic manager will, therefore, determine the speed of the airship not by dividing 2500 km by 24 (= 106 km/hr.), but by 36 (= 70 km/hr.). Any further shortening of the flight time would, as a rule, be uneconomical, since either the landing would have to be made in the night or an unfavorable change in the starting time would have to be made, which would work against the principle that travel by day is lost time. If, however, the flight time approximates an even multiple of 12 hours, every shortening is a gain until an uneven multiple of 12 hours is reached. (We are at first making no allowance for the time required for landing maneuvers, embarking and debarking, loading and unloading; nor for the prevailing direction of the winds.)

In Fig. 11, it is attempted to represent the time saving, the plain lines (I) giving the saving for an even multiple of 12 hours and the dash line (II) for an uneven multiple of 12 hours.

Flight duration and requisite number of airships.- Speed reduction does not lead indefinitely to expense reduction. If a given load is to be carried daily, i.e., if a definite flight schedule is to be maintained, the flight duration, with allowance for the stops at the end stations, determines the number of airships necessary for maintaining the schedule. Now, since the capital investment depends on the number of airships, it may happen, under certain conditions, that a reduction in the speed (i.e., in the simple operating expenses) will so greatly increase the investment, as to make the total outlay greater instead of smaller.

It is assumed that, in a regular daily traffic over a given distance, each airship should have a rest of 12 hours between trips. If it is further assumed that the departures are to take place only in the morning or evening, we obtain, by successive increments of 12 hours in the flight duration, the schedules represented by Fig. 12. In (a) the flight duration is 12 hours. In order to avoid loss of time (i.e., to utilize the night time for traveling), the departures from both A and B must take place in the evening, the intervening hours being employed in overhauling, unloading, loading, etc. A regular daily traffic in both directions can thus be maintained with only two airships.

b) If the flight duration is 24 hours, the departures from one place will take place in the morning and from the other in the evening and three airships will be required.

c) If the flight duration is 36 hours, the departures from both places will be in the evening and four airships will be required.

d) If the flight duration is 48 hours, the departures from one place will occur in the evening and from the other in the morning, five airships being required.

Herefrom the following rules can be deduced:

a) If the flight duration ($S \div 3600\ v$) is an odd multiple of 12 hours, the departures from both places will occur in the evening;

b) If the flight duration is an even multiple of 12 hours, the departures from one place will occur in the evening and from the other in the morning.

The required number n of airships is

$$n = \frac{S}{12 \times 3600\ v} + 1 \tag{40}$$

Flight duration and capital investment.- If traffic service is desired over a given line and if the characteristics of the airships are adapted to all the conditions, especially to the desired speed, then the required capital will depend not on the distance, but on the time required to make it. The flight time determines the required number of airships which, in turn, determines the requisite arrangements at the terminal stations. Furthermore, the number of intermediate stations, whether designed as regular stopping places or only for emergency landings, depends on the flight time and not on the distance. This is due to the fact that all considerations of this nature are based on the time. For regular intermediate stops, the time lost must be considered. The installation of emergency fields is based on the consideration as to how long a time a damaged airship can continue flight, since S alone is never conclusive, but always the quotient of $S \div v$, i.e., the time t.

All assumptions concerning the requisite capital are naturally somewhat arbitrary and hence debatable. It is, however, necessary to establish certain basic principles, in order to be able to proceed.

If the principle is adopted that a reserve airship must be able to reach the airship to be replaced within 24 hours after leaving its own hangar, we come to the conclusion, under consideration of equation (40), that there should be one reserve airship for every group of three airships or fraction thereof. Of hangars and mooring masts there must be erected a total number equal to the number of airships on hand. On long routes there may be about twice as many masts as hangars provided. On the further assumption that one gas plant is sufficient for three airships, including the reserve airships, we arrive at the figures given in Table VIII.

Table VIII.

Flight duration hr.	No. of airships in service	in reserve	Total	Main stations. Hangars	Mooring masts	Intermediate stations Gas plants.	Intermediate landing fields with masts.
12	2	1	3	2	2	2	–
24	3	1	4	2	3	2	–
36	4	2	6	3	3	2	1
48	5	2	7	3	4	3	1
60	6	2	8	4	4	3	2
72	7	3	10	4	5	4	2
84	8	3	11	4	5	4	3
96	9	3	12	5	6	4	3
108	10	4	14	5	6	5	4
120	11	4	15	5	7	5	4

It is better to disregard, at first, the effect of the size of the airships on the cost of the enterprise. So long as the size of the airships is not established, we could only make rough estimates, which would have to be subsequently corrected. This omission seems all the more permissible, because in small enterprises, their subsequent expansion would be contemplated from the first.

The following estimates are based on various publications, e.g., Pratt's "Commercial Airships," Scott's "Lectures," etc. They are presented only in small groups, since it is not intended to give accurate figures (almost impossible under present conditions), but only a calculation method.

Assumed cost of plant:

One hangar with workshops, etc.	$1,250,000
Land	250,000
Total cost of a hangar station	1,500,000
One mooring mast	150,000

Land	$100,000
Total cost of mast station	250,000
One gas plant	600,000

15% of the total cost of the enterprise must be raised by the management, as follows:

Interest on invested capital	5%
Sinking fund	7%
Upkeep	2%
Insurance	1%
Total	15%

From the above figures we obtain, by means of Table VIII, the estimates given in Table IX.

Table IX.

Flight duration hr.	Hangar $	Mooring mast $	Gas plant $	Total $	Annual 15% $
12	3,000,000	500,000	1,200,000	4,700,000	705,000
24	3,000,000	750,000	1,200,000	4,950,000	742,500
36	4,500,000	1,000,000	1,200,000	6,700,000	1,005,000
48	4,500,000	1,250,000	1,800,000	7,550,000	1,132,500
60	6,000,000	1,500,000	1,800,000	9,300,000	1,395,000
72	6,000,000	1,750,000	2,400,000	10,150,000	1,522,500
84	6,000,000	2,000,000	2,400,000	10,400,000	1,560,000
96	7,500,000	2,250,000	2,400,000	12,150,000	1,822,500
108	7,500,000	2,500,000	3,000,000	13,000,000	1,950,000
120	7,500,000	2,750,000	3,000,000	13,250,000	1,987,500

In addition to the annual costs of the simple capital investment, there comes a series of yearly outlays more or less closely related to the former. These also can be only approximately estimated.

They are most conveniently expressed in percentages of the annual construction costs:

Operation and upkeep	50%
Advertising	15%
Office maintenance	20%
Interest on operating capital	15%
Total	100%

This means that the annual costs in Table IX must be doubled, in order to include the above items.

All these expenses are independent of the length of the routes and of the service rendered. They depend only on flight time.

Operating costs.- The costs of operating the airships themselves fall into two groups. The first group is independent of whether and how much the airships are actually used, while the second depends only on the distance flown during the year at a given speed.

It has been shown that the size of the airship is determined by the desired pay load, the distance between stations and the speed. The cost of the airships and their annual upkeep are closely related to their size. We will not err greatly if we call the cost of the airship proportional to its size. Calculations made in other places admit this assumption. Corrections can be readily made in special cases. $1,500,000 may be considered a moderate price for an airship of 100,000 m^3 gas capacity. If we assume that the cost of the crew is likewise proportional to the size of the airship, the first group of annual expenses may be expressed in percentages of the original purchase price, as follows:

Interest on cost of airship		5.0%
Amortization		25.0%
Upkeep		12.5%
Insurance		7.5%
Crew		4.0%
	Total	54.0%

For further computation, these values must be connected with the size and number of airships.

The simple operating expenses depend on the fuel consumption. If we compute the gasoline and oil together at 19 cents per kilogram, call the hydrogen consumption in cubic meters equal to the weight of the fuel consumed and appraise its value, with consideration of other losses, at the rather high price of 7 cents per kilogram of fuel consumed, we obtain a total cost of all the materials consumed amounting to 26 cents per kilogram of fuel consumed. (Under certain conditions, the flight altitude would necessitate a correction of the gas consumption.) The ratio of the operating costs to the engine power and distance flown (or flight time) is thus directly determined.

<u>Total annual cost</u>.- The total cost thus consists of the annual construction costs X, the annual cost of the airships and

their upkeep Y and the annual cost of consumable materials (gasoline, oil and hydrogen) Z. If we calculate 360 days per year for the traffic in both directions, we obtain a total distance of 360 S × 2 = 720 S. The transportation service is accordingly 720 Q S at the desired period v.

If, according to Table IX, the annual construction costs $X = f(S/3600\ v)$ are plotted in a diagram (Fig. 13), we obtain a nearly straight line. The construction costs must be doubled, as already explained, and X may therefore be replaced by

$$X = 2\ \omega_1\ \frac{S}{3600\ v} + 2\,\Omega_1 \qquad (41)$$

With the values from Fig. 13, we obtain $\omega_1 = 13070$ and $\Omega_1 = 540000$. Hence

$$X = \frac{26140}{3600} \times \frac{S}{v} + 1080000 \qquad (42)$$

The annual expenses for one airship were found to be 54% of its value. It was further stated that we must calculate on one reserve airship for every three scheduled airships. The annual expenses for the reserve airships are about the same as for the others, since crews and supplies must likewise be held in reserve for them. The annual expenses must therefore be increased 1/3, thus making 72% of the value of each airship to be taken into the calculation. Hence we have $0.72\,\omega_2$ V dollars for each airship or, with $\omega_2 = \$15/m^3$ on the basis of a previous assumption, 10.8 V dollars.

Since, however, according to equation (40), a total of $n = \frac{S}{12(3600\ v)} + 1$ airships is required, the cost of the airships is

$$Y = 10.8\ V \left(\frac{S}{12 \times 3600\ v} + 1\right) \tag{43}$$

The annual cost of the consumables is

$Z = \omega_3\ B_j$ or (with ω_3 = \$0.26 per kg)

$Z = 0.26\ B_j$.

If the consumables for a single trip are represented, as before, by B, we have $Z = \$0.26 \times 720\ B$ or, since, according to equation (31), $B = \frac{e\ c\ V^{2/3}\ S\ v^2}{3600}$,

$$Z = \frac{0.26 \times 720 \times e\ c\ V^{2/3}\ S\ v^2}{3600} \tag{44}$$

At this point it would have been more logical to employ the factor b instead of e (See definition). It is more convenient, however, to retain e, since the same expression occurs again in the further calculation. In view of this, the basic price of \$0.26 per kg for the consumables should have been diminished in the ratio $b : e$. If, however, the higher value is retained, we may consider it as including the losses due to trial and idling runs, wind, etc.

The ordinary annual expenses are therefore

$$\Phi = X + Y + Z \tag{45}$$

and

$$\Phi = \frac{2\,\omega_1\,S}{3600 v} + 2\,\Omega_1 + \omega_2\omega_4 V \left(\frac{S}{12\times 3600 v} + 1\right) + \frac{\omega_3\,\omega_5\,e\ c\ V^{2/3}\ S v^2}{3600} \qquad (46)$$

or, with the coefficient values employed wherein, as before, $e = 2.5 \times 10^{-1}$ and $c = 3 \times 10^{-5}$,

$$\Phi = \frac{26140}{3600}\,\frac{S}{v} + 1080000 + 10.8\ V\left(\frac{S}{12 \times 3600\ v} + 1\right) + \frac{0.26 \times 720 \times 2.5 \times 3\ V^{2/3}}{3600 \times 10^6}\,S$$

The economical speed.- If the chosen speed succeeds in reducing the expenses to the minimum, this speed is the economical speed for the enterprise. It is much more important than the speed of minimum fuel consumption, which was discussed in a preceding section. It is of primary importance, while the speed of minimum fuel consumption can enter the problem only for an individual flight and even then only under certain conditions.

In the further process $\frac{d\Phi}{dv} = 0$ is first formed:

$$\frac{d\Phi}{dv} = -\frac{26140}{3600 v^2} - \frac{10.8\ V}{12\times 3600\ v^2} + \frac{0.26\times 720\times 2.5\times 3\times 2\ V^{2/3}\ v}{3600\ \times 10^6} = 0$$

whence

$$v^3 = 10^6 \times \frac{26140 + 2.9\ V}{2808\ V^{2/3}} \qquad (48)$$

This equation contains the condition for the minimum, since $f''\ v > 0$. From it we obtain homogeneous values of v and V. Table X contains a series of such values. According to

Table X.

V m^3	26140 +0.9V	$V^{2/3}$	v^3	v(m/s)
10,000	35,140	464	26,960	29.99
20,000	44,140	737	21,330	27.73
40,000	62,140	1169	18,930	26.65
60,000	80,140	1533	18,620	26.50
80,000	98,140	1856	19,010	26.60
100,000	116,140	2154	19,200	26.78
120,000	134,140	2433	19,630	26.98
140,000	152,140	2696	20,100	27.19
160,000	170,140	2947	20,560	27.40
180,000	188,140	3189	21,010	27.59
200,000	206,140	3420	21,460	27.79
250,000	251,140	3968	22,540	28.25
300,000	296,140	4481	23,540	28.66

these, the curve of the determining equation becomes

$$v = 10^2 \sqrt[3]{\frac{26140 + 0.9\ V}{2808\ V^{2/3}}} \qquad (49)$$

as plotted in Fig. 14.

The second definitive equation, available for v and V, is No. (33)

$$a\ V = k\ V^{4/5} + i\ c\ V^{2/3}\ v^3 + \frac{e\ c\ V^{2/3}\ S\ v^2}{3600} + Q.$$

It was assumed at the outset that Q and S were of given magnitudes. Hence, according to Fig. 10, the corresponding curve III represents the second place for v and V. If this curve and the curve in Fig. 14 are reduced to the same scale, they intersect at a point which must fulfill the following conditions (Fig. 15):

1. The values of v and V are so coordinated that, with given Q and S, the size of the airship is the maximum for the desired speed (equation (33).

2. The expenses of the whole enterprise are minimum at the speed v (equation (49)).

Thus the desired optimum, the economic speed for the enterprise, is found.

Even if a method is successfully developed with whose help we can find the economic speed of an airship for commercial purposes, it would still not necessarily follow that the calculated speed could be actually attained or that it would be practicable. It might be so great or so small as to be out of the question for the traffic. It was, moreover, possible that even slight changes in V might produce great changes in v. In either event, only theoretical importance could be attached to the economic speed.

In fact, the rapidity of change was so great in the first portion of the curve (Fig. 14), up to a gas capacity of about 20,000 m^3 as to render it difficult to harmonize the volume and speed at just the best values. This branch of the curve is, however, entirely irrelevant for practical airship flight. Since nowadays airships of less than 75,000 m^3 are regarded as unsatisfactory for actual service, only volumes of 75,000 - 250,000 m^3 are of interest. For these volumes, the nearly straight, gradually ascending curve indicates corresponding speeds exceptionally suited to all other aspects of a commercial enterprise. With 26.6 to 28.25 m/s (i.e., with a mean speed of about 27.5 m/s) the economic speed is just high enough to make an airship a successful instrument of rapid transportation. Moreover, this speed is low enough to be readily attained. Furthermore, deviations from the computed volume, which might be desirable for other reasons, produce only such slight changes in the economic speed as to be of no practical importance. Hence the result is of actual value, since it proves to be practically utilizable.

Considerations regarding the adopted assumptions.- During the development of the computation method, a long series of assumptions was necessary. It is therefore appropriate to consider what effects certain wrong assumptions might have on the results. There are also a few additional factors to be considered, concerning which unexplained assumptions were made.

Costs.- If all the costs should change uniformly (e.g., if they should all double), the result would not be affected. If the construction costs should increase, the economic speed would increase with the third root of the factor of change. The reverse would occur, if the basic prices of the consumable should rise.

Pay load, distance, dead load, weight of each engine, reserves.- Any increase in these factors would increase the volume of the airship and hence the economic speed.

Specific fuel consumption.- If this increases, the volume increases according to equation (33) and v decreases according to equation (49), which takes account of the specific fuel consumption in the factor 2808. The effects of these changes offset one another, at least partially.

Wind.- In Part II of this article, the unexplained assumption was made that the airspeed and ground-speed were equal, i.e., that $w = 0$. Such, of course, is never the case. The effect of the wind generally requires a greater airspeed than ground-speed, the result being to increase the fuel consumption. Hence it is well to allow for the effect of the wind in the corresponding valuation of e. In this event, v in equation (46) is replaced by the ground-speed u and equation (49) then gives the value of u which reduces the expenses to a minimum.

Ground-speed.- Any desired change in the ground-speed can be, to a certain degree, offset by changing the duration of the stops.

Schedule.- Any fundamental change in the schedule first necessitates a determination of the requisite number of airships.

Time lost in landing.- On long trips this will not affect the required ground-speed, but on short trips it will necessitate an increase in the ground-speed.

Low barometer, crossing mountains.- Allowance can be made by changing a or by introducing special factors. Hereby V is increased and sometimes also v.

Example.- It is desired to establish airship traffic over a distance of 4000 km between the places A and B. It is required to carry 50 passengers daily in each direction and to provide them with comfortable cabins. A cabin allowance of 320 kg is made for each passenger, in addition to 80 kg for his own weight. An allowance of 2.5 metric tons is made for provisions for passengers and crew, 25 metric tons for freight and 10% of the latter for arrangements for stowing the freight.

Solution.- The total pay load comprises:

50 passengers,	(80 kg each) -	4000 kg
Cabin weight,	(320 kg per passenger) -	16000 "
Freight,		25000 "
10% storage devices,		2500 "
Provisions,		2500 "
	Total	50000 "

Fig. 16 gives both curves of equation (33) for $Q = 50$ t and $S = 4000$ km and also the curve $v = f(V)$ derived from the corresponding values. The latter curve is intersected by the curve $u = f_1(V)$ plotted on the same scale according to equation (49). The intersection point gives:

Ground-speed	26.5 m/s
Volume of airship	75000 m³.

The ground-speed increased, over that called for by the schedule, to 27.5 m/s, thus making the flight time 40 hours. In order to show how further factors can be taken into account, the following conditions are assumed:

A wind of 8 m/s mean velocity at an angle of 20° to the direction A B;

Maximum wind velocity on the course, 28 m/s;

Mean barometer at A, 750 mm; at B, 760 mm;

Mean temperature at A, 10°C; at B, 15°C;

At 1000 km from A, a mountain range to be crossed at an altitude of 2000 m.

Solution.- Taking into account the maximum wind velocity, the airship would attain a maximum speed of $v_1 = 36$ m/s. The mean wind velocity of 8 m/s at an angle of 20° requires, for the groundspeed of 27.5 m/s, an airspeed of

$$v = \sqrt{u^2 + w^2 \pm 2\,u\,w\,\cos 20^\circ}$$

to $$v_2 = 35.1 \text{ m/s}$$

and $$v_3 = 20.2 \text{ m/s}$$

The lift coefficient is

$$a_A = 1.17 \times \frac{750}{760} \times \frac{273}{283} = 1.11 \text{ at A}$$

and $$a_B = 1.17 \times \frac{273}{288} = 1.12 \text{ at B}$$

The smaller value must be employed in the computation. Account may be taken of the altitude, either by using the logarithmic altitude formula or by diminishing the lift 1% for each 80 m increase in altitude. Thus an altitude increase of 2000 m corresponds to a lift decrease of 25%. This is increased by the fuel consumption over a distance of 2000 km, during which the minimum airspeed is to be assumed, i.e., $v_3 = 20.2$ m/s; further by the dynamic lift which may be assumed, from experience, to be 0.05 a V kg (Its maximum is about 0.1 a V kg. The obtention of this value, however, so greatly reduces the horizontal speed that it should be utilized only in case of emergency) and lastly by discharging ballast R. We thus obtain

$$R = \frac{h\ 0.01\ a\ V}{80} - \frac{b\ c\ V^{2/3} S_x v^2}{3600} - 0.05\ a\ V$$

in which h denotes the flight altitude an S_x the distance to the point where the altitude h must be attained.

Reserve fuel can be taken into account by increasing the value of e to $e_1 = 0.275$ and to the higher airspeed $v_2 = 35.1$.

Equation (33) now acquires the form

$$a_A\ V = k\ V^{4/5} + i\ c\ V^{2/3}\ v_1^3 + \frac{e_1\ c\ V^{2/3}\ S\ v_2^3}{3600} + Q + 0.25\ a_A\ V$$
$$- \frac{1000}{4000}\ \frac{b\ c\ V^{2/3}\ S\ v_3^2}{3600} - 0.05\ a_A\ V.$$

The last three terms correspond to the ballast discharged. Care must be exercised that the values

$$0.25\ a_A\ V - \frac{b\ c\ V^{2/3}\ S\ v_3^2}{4 \times 3600} - 0.05\ a_A\ V$$

is introduced only so long as it is positive. As soon as it becomes negative, these three terms must be dropped, since otherwise the airship would be forced to rise from the ground by dynamic force alone. It is further necessary, so that the dynamic lift can be subsequently offset statically, that

$$0.05\ a_A\ V < \frac{b\ e\ V^{2/3} v_3^2}{3600}\ (S - S_x).$$

In the present instance the terms are retained and the equation accordingly becomes

$$0.8\ a_A\ V = k\ V^{4/5} + i\ c\ V^{2/3} v_1^3 + \frac{c\ V^{2/3} S}{3600}\left(e_T\ v_2^2 - \frac{b}{4} v_3^2\right) + Q$$

or, with the adopted numerical values,

$$0.8 \times 1.11\ V = 3\ V^{4/5} + 4 \times 3 \times 10^{-5} \times 36^3\ V^{2/3} +$$

$$\frac{3 \times 10^{-5} \times 4 \times 10^6\ V^{2/3}}{3600}\left(0.275 \times 35.1^2 - \frac{0.2 \times 20.2^2}{4}\right) + 50000$$

$$0.888\ V - 3\ V^{4/5} - 16.2\ V^{2/3} = 50000$$

from which we obtain: $V = 158000\ m^3$.

The other conditions were intèntionally so selected as to give a large difference in comparison with the first-found airship volume and, indeed, in order to demonstrate that the effect on the economic speed is only slight. From Fig. 16 the economic ground-speed is found to be $u = 27.5$ m/s, i.e., just as great as it was originally adopted for the schedule.

The scheduled speed of 27.5 m/s gives a flight duration of about 40 hours for 4000 km. If stops of only 8 hours are made at the terminal stations, the schedule would correspond to Fig. 12c, requiring four airships in operation and two in reserve.

On the basis of the previous assumptions, the enterprise would cost as follows:

Capital investment:

3 hangars at	\$1,500,000	\$4,500,000
3 masts "	250,000	750,000
2 gas plants at	600,000	1,200,000
6 airships "	2,400,000	14,400,000
Operating capital		3,000,000
	Total	\$23,850,000

Annual expenses:

2 x 15% of the construction cost		\$ 1,935,000
54% of the airships' cost		7,776,000
Consumables		3,140,000
Gas loss from altitude		1,372,000
	Total	\$14,223,000

[The cost of the consumables was computed from

$$\frac{0.26 \times 360 \times b \times c\, V^{2/3} S\, (v_2^2 + v_3^2)}{3600}$$

and the value of the gas loss (due to altitude) from

$$0.07 \left(720 \times 0.25\, a_A V - \frac{360\, b\, c\, V^{2/3} S (v_2^2 + v_3^2)}{3600}\right)]$$

The sum of \$14,223,000 is the minimum amount of the annual expenses capable of carrying on the enterprise under the given conditions.

The receipts, for covering the above costs, were computed as follows:

Each trip required:

50 passengers with a total weight, including cabin, of 50 × 400	20,000 kg
Freight, including weight of storerooms,	27,500 "
Provisions,	2,500 "

The weight of the provisions may be assumed as 4/5 for the passengers and 1/5 as freight, by dividing the crew's provisions of 2/5 of the whole equally between the two pay classes, passengers and freight. Thus we obtain as pay load:

50 passengers at		440 kg	= 22,000 kg
25,000 kg	×	1.12 kg	= 28,000 "
		Daily one-way total	50,000 "

The total weight carried annually is thus 50000 kg × 720 = 36,000,000 kg and the cost per trip is \$14,223,000 ÷ 36,000,000 = \$0.40 per kilogram, i.e., \$176 for each passenger and \$0.448 per kg of freight. Reduced to unit distance, this gives:

Per passenger-kilometer, \$0.044

Per freight-kilometer, 0.000112 per kg.

All receipts above these figures are pure profit.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

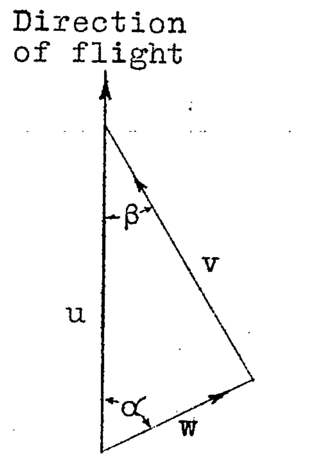


Fig. 1

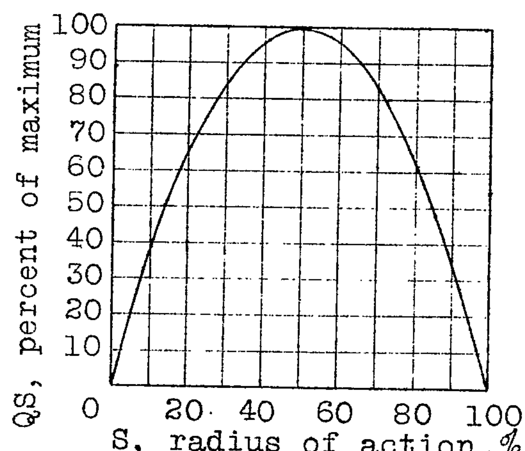


Fig. 2

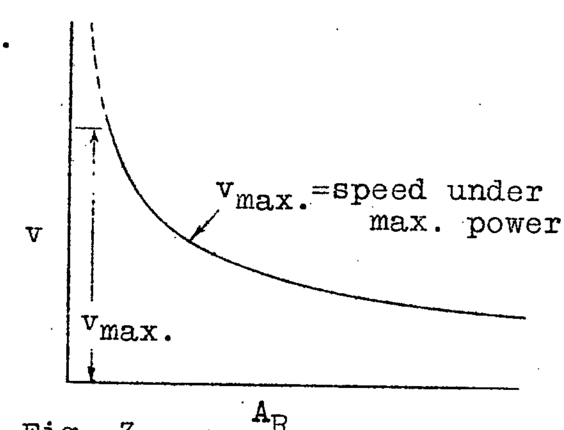


Fig. 3

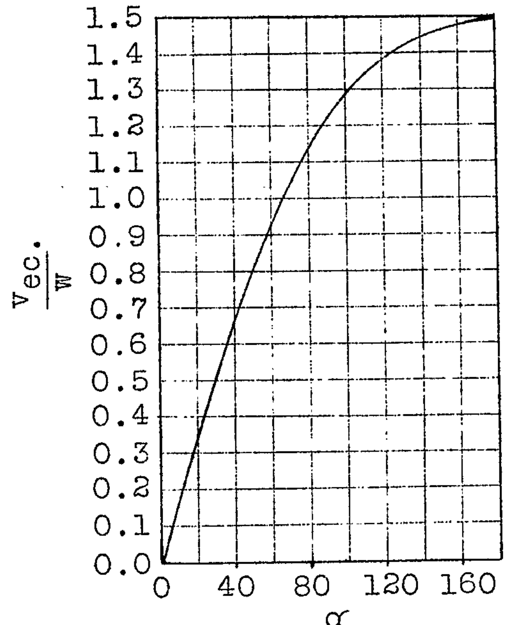


Fig. 4

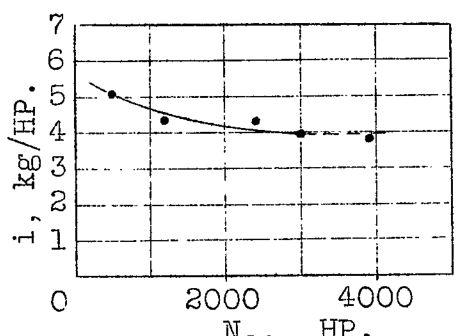


Fig. 5

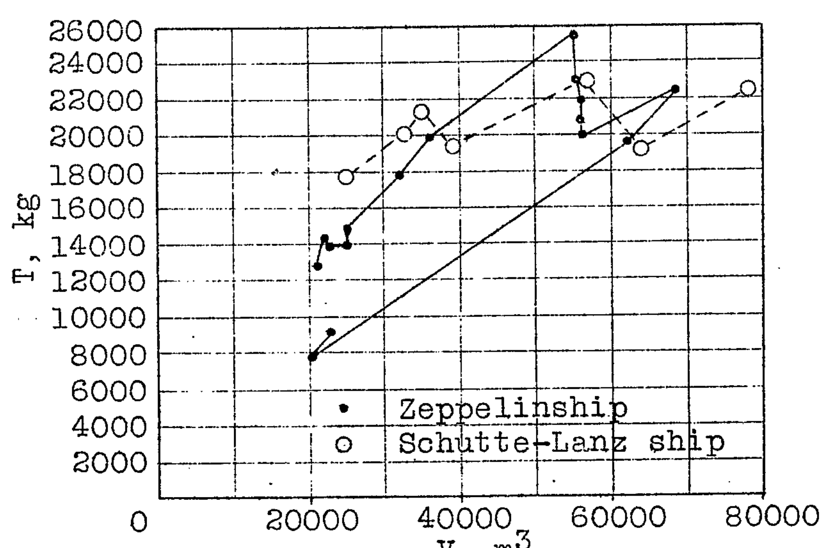


Fig. 6

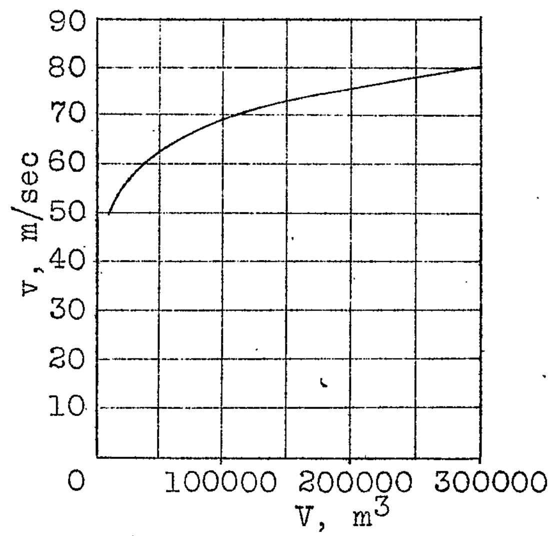
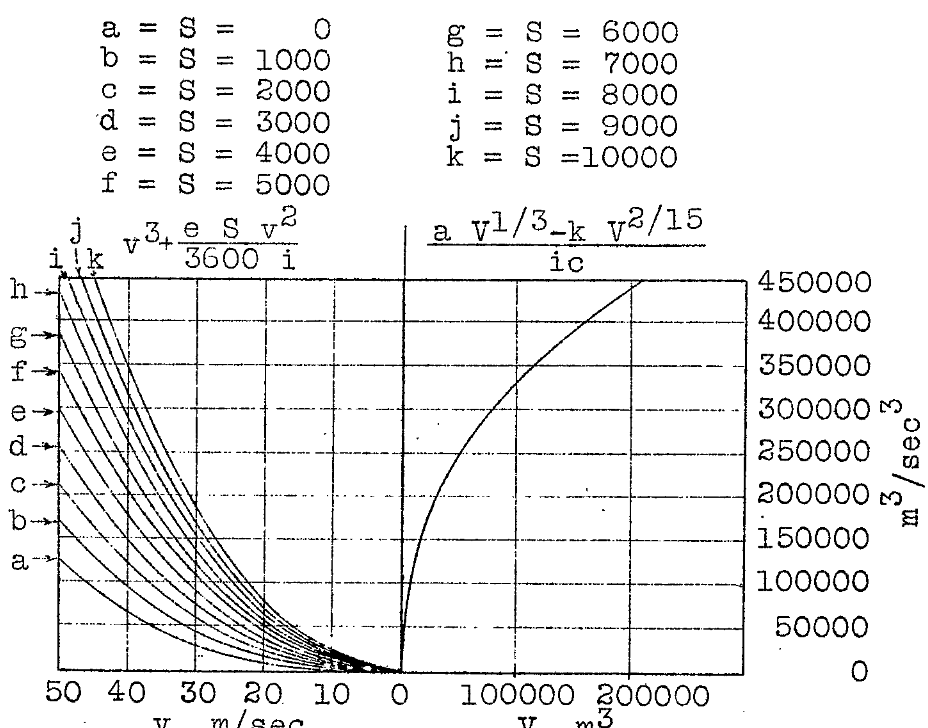

90
80
70
60
50
40
30
20
10
0
v, m/sec
100000 200000 300000
V, m3

Fig. 7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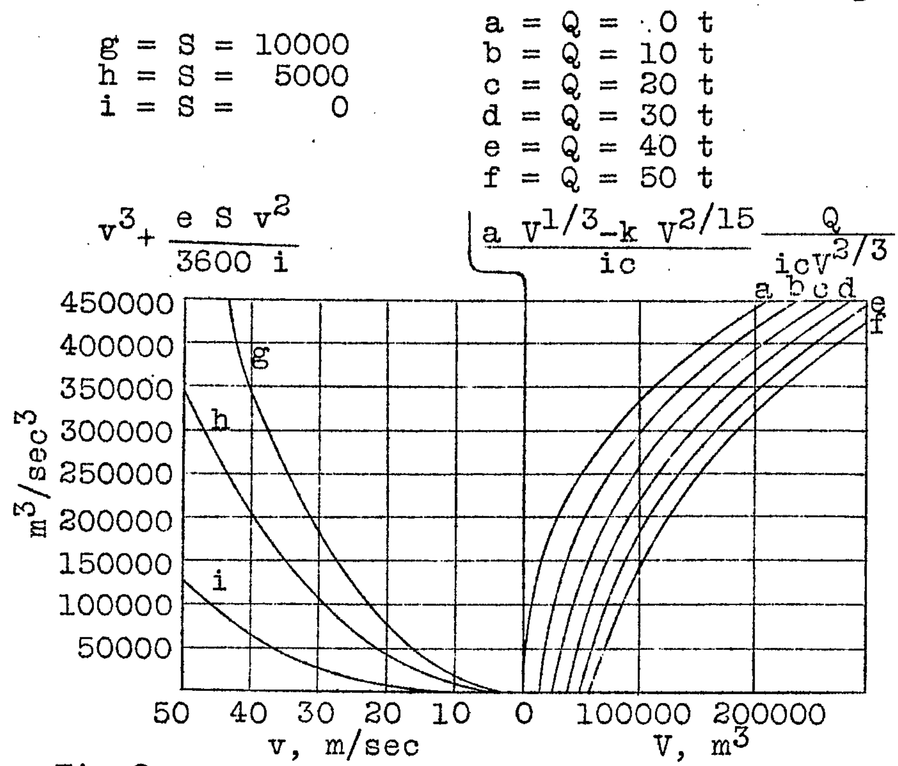
b = S = 1000
c = S = 2000
d = S = 3000
e = S = 4000
f = S = 5000
g = S = 6000
h = S = 7000
i = S = 8000
j = S = 9000
k = S =10000
$v^3 + \frac{e\ S\ v^2}{3600\ i}$
$\frac{a\ V^{1/3} - k\ V^{2/15}}{ic}$
h
g
f
e
d
c
b
a
i
j
k
450000
400000
350000
300000
250000
200000
150000
100000
50000
0
m3/sec
50 40 30 20 10 0
v, m/sec
0 100000 200000
V, m3

Fig. 8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h = S = 5000
i = S = 0
a = Q = 0 t
b = Q = 10 t
c = Q = 20 t
d = Q = 30 t
e = Q = 40 t
f = Q = 50 t
$v^3 + \frac{e\ S\ v^2}{3600\ i}$
$\frac{a\ V^{1/3} - k\ V^{2/15}}{ic} - \frac{Q}{icV^{2/3}}$
450000
400000
350000
300000
250000
200000
150000
100000
50000
m^3/sec^3
50 40 30 20 10 0 100000 200000
v, m/sec
V, m^3
a b c d e f
g
h
i

Fig.9

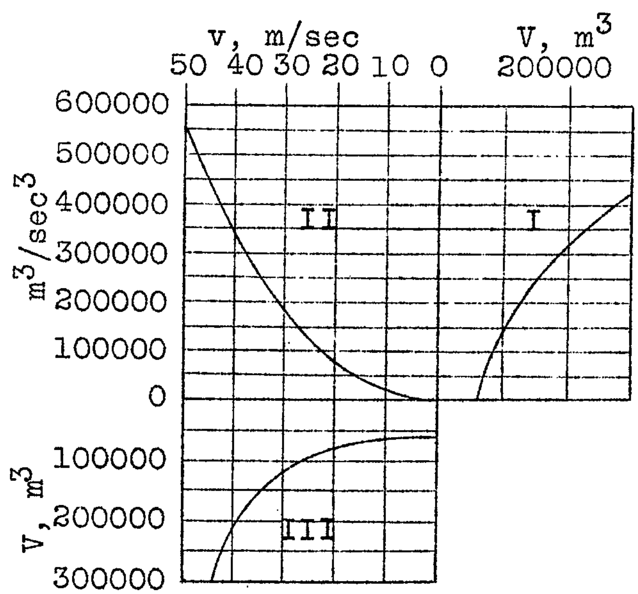

v, m/sec
50 40 30 20 10 0
V, m^3
200000
600000
500000
400000
300000
200000
100000
0
m^3/sec^3
100000
200000
300000
V, m^3
I
II
III

Fig.10

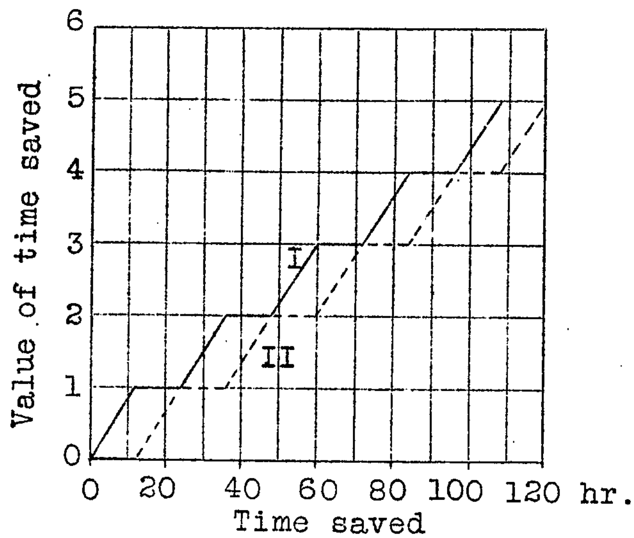

Value of time saved
0
1
2
3
4
5
6
I
II
0
20
40
60
80
100
120 hr.
Time saved

Fig.11

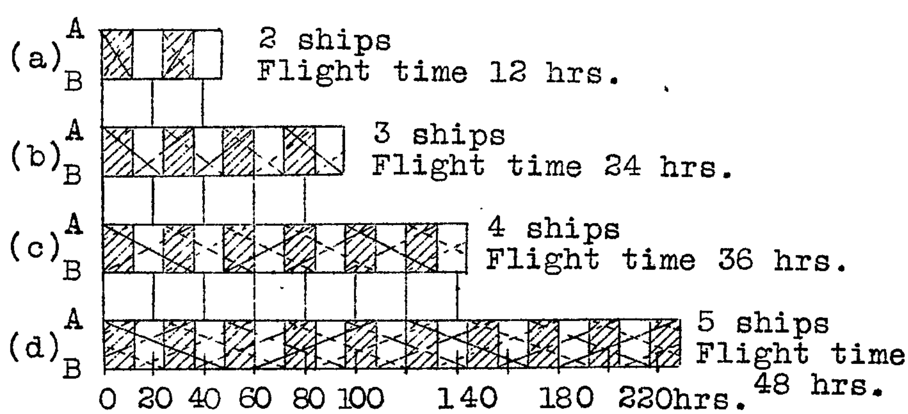
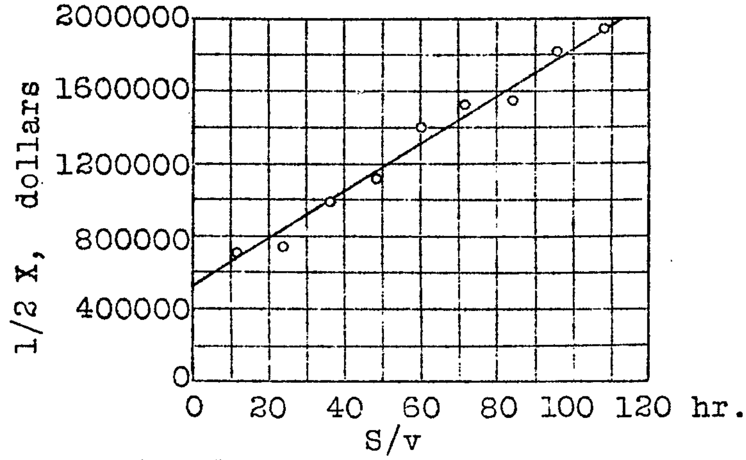
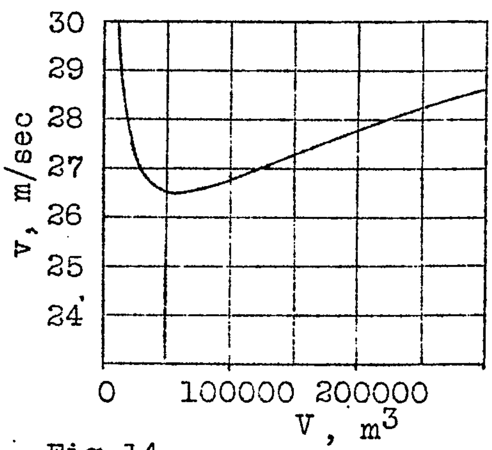

(a) A B
2 ships
Flight time 12 hrs.
(b) A B
3 ships
Flight time 24 hrs.
(c) A B
4 ships
Flight time 36 hrs.
(d) A B
5 ships
Flight time 48 hrs.
0 20 40 60 80 100 140 180 220hrs.

Fig.12



2000000
1600000
1200000
800000
400000
0
1/2 X, dollars
0 20 40 60 80 100 120 hr.
S/v

Fig.13



30
29
28
27
26
25
24
v, m/sec
0 100000 200000
V , m3

Fig.14